

UD Computational Science Day 2006

February 14, 2006

Why It Is Safe to Use Gaussian Elimination with Partial Pivoting

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Abstract

The growth factor of a matrix quantifies the amount of potential error growth when a linear system is solved by Gaussian elimination with partial pivoting. For an $n \times n$ matrix, the growth factor has a maximum value of 2^{n-1} indicating a possible loss of $n - 1$ bits of precision in a computed solution. In practice, the occurrence of matrices with exponentially large growth factors is extremely rare. Due to the rarity, it is exceedingly difficult to construct an approximate probability density function for large growth factors experimentally. To overcome this difficulty, we carried out a multicanonical Monte Carlo simulation with a Metropolis-Hastings sampler. The multicanonical Monte Carlo simulation iteratively biases Monte Carlo simulations. As the number of iterations increases, the sampling strategy employed gives each region of attainable growth factors the same relative importance. This allows meaningful statistics about the tail of the pdf to be compiled for large growth factors. Our results, found on an Opteron cluster using the Distributed Computing Toolbox in MATLAB, suggest that the encounter of an 8×8 matrix with a growth factor of 25 is on the order of a once-in-the-age-of-the-universe event.